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# MATHEMATICS OF ORBIT DETERMINATION

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# MATHEMATICS OF ORBIT DETERMINATION

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Joseph W. Siry

## INTRODUCTION

Orbit determination for artificial satellites and spacecraft has developed upon the foundations laid by university research workers over the centuries. Elaborate theories have been developed to cope with the problem of determining the orbits of the planets and of their natural satellites. Modifications of these theories have been developed in order to make them applicable to some of the special problems associated with the orbits of artificial satellites.

The fundamental problem of orbit determination is that of comparing theories such as these with observational data obtained from satellite tracking stations. Various aspects of this problem are indicated in Figure 1. Preparations for orbit determination for the satellites to be launched during the International Geophysical Year were based upon the assumption that the principal perturbations would be due to the earth's oblateness and its atmosphere.

Satellites are slowed down and lose energy due to the aerodynamic drag force which is encountered primarily in the neighborhood of perigee. When a satellite loses energy, its orbital period decreases. Thus, the observed decrease in the period of a satellite orbit can be interpreted in terms of atmospheric density in the neighborhood of perigee. It was thought prior to the IGY that the atmospheric density varied with height, but not significantly with latitude, longitude or time. Measures of air density were available from the sounding rocket programs pursued prior to the IGY. The IGY orbits were to have perigee altitudes of about 200 miles and apogee altitudes of about 1400 miles. This perigee height was selected since it would permit the study of atmospheric density in the region near 200 miles altitude, which was above the level for which the sounding rocket data were available.

The satellites were designed to be spherical. The effective cross-sectional area

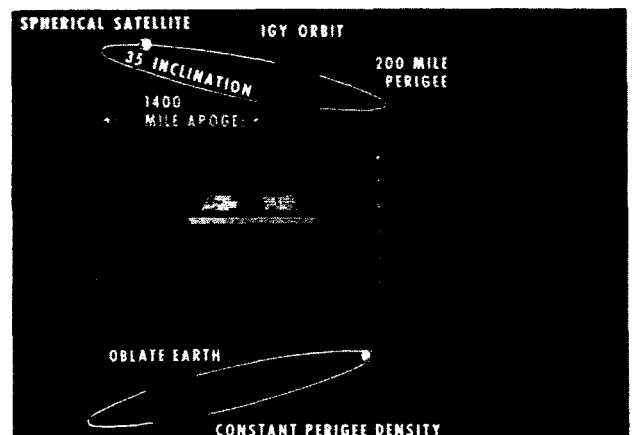


Figure 1--Orbit determination.

presented to the aerodynamic flow would thus be constant and predictable, even though the satellite's attitude or orientation would not necessarily be known.

The inclination of  $35^\circ$  was the maximum which could be achieved safely with the launching rocket vehicle systems then contemplated.

Orbit determination systems have been called upon to do many things since the space age began. There is always interest initially in the early estimates of the principal orbital parameters such as the period, the perigee and apogee heights, and the inclination. In addition, it is necessary to provide accurate predictions for the tracking and data acquisition stations in order to insure that additional tracking and telemetry information will continue to flow into the data gathering and processing systems. General viewing predictions also provided for satellites which can be seen visually by observers around the world. It was also necessary to furnish detailed ephemerides for the experimenters whose instruments operated in the IGY scientific satellites. Information consisting of the longitude, the latitude and the altitude of the satellite at each minute of its active radio lifetime is normally furnished to each scientist conducting an experiment with satellite-borne instrumentation.

It was anticipated that analysis of satellite orbital tracking data would yield new information concerning air densities and the shape of the earth. In particular, it was hoped that it would be possible to obtain more accurate measures of the oblateness of the earth.

Orbit determination includes the analysis of the tracking system which is used to furnish the data upon which the ephemerides and analyses are based. Accordingly, it was anticipated that important information about the tracking system would be derived from the orbit determination program.

These, then, were and still are the principal aspects of the science of orbit determination. As the space programs have become more sophisticated and complex, a number of facets of orbit determination have developed correspondingly.

A number of surprises were in store for those analyzing orbital data as they sought to glean geophysical information. For example the

very first satellite showed that the atmospheric density was greater than previously supposed by a factor of the order of five. The oblateness of the earth was found to be significantly different from the estimates obtained on the basis of geodetic information. A number of additional geophysical discoveries were made. Some of these are indicated in Figure 2. For example, it was learned from orbital information that the earth is actually pear shaped. Quantitative information has now been obtained about a number of the zonal harmonics

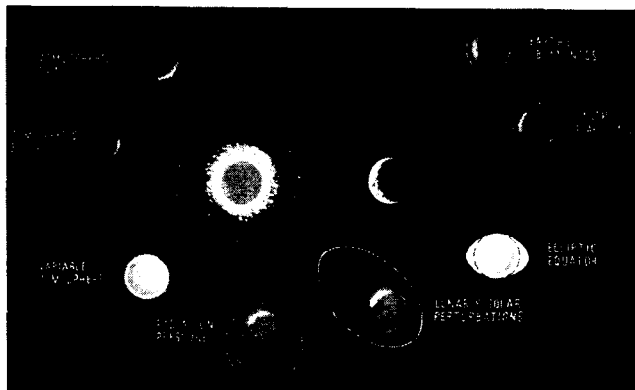


Figure 2—Environment factors affecting orbit determination.

of the earth's gravitational field. Further studies have also begun to shed more light on the ellipticity of the earth's equator. A number of new and important discoveries were made concerning the earth's atmosphere, using orbital data. It was found that the atmosphere actually bulges on the side of the earth which faces toward the sun. The bulge effect is greatest in longitudes that correspond to the early afternoon hours. This was discovered by observing that the satellite orbital period decrements, and hence the corresponding atmospheric densities, increased as the perigees of elliptical satellite orbits moved from the dark side of the earth around toward the bulge. In addition, it was observed that the atmospheric density actually increased rather sharply shortly after certain types of solar flares were observed to occur on the sun. This is indicated in Figure 3. In addition to specific changes in atmospheric density in response to individual solar events, a general correlation was found to exist between solar activity and atmospheric density at high altitudes. The active regions on the sun are not homogeneously distributed with respect to longitude. They occur more frequently in some solar longitudes than in others, for example. These clusters of active regions rotate with the sun, whose rotational period is 27 days. It has been found that the atmospheric density at high altitudes varies with a period of 27 days, and that the correlation with solar activity is marked. This correlation is more pronounced in the neighborhood of the atmospheric bulge. The pressure of radiation upon a satellite is small and is often considered to be negligible. Acting over a long period, however, it does produce a detectable effect,



Figure 3—Variable atmospheric drag effects on satellite orbits.

even upon the orbit of an ordinary satellite. Radiation pressure can result in a significant perturbation of the orbit in the case of a low-density satellite such as Echo. Lunar and solar perturbations can also be observed, even on the orbits of close earth satellites.

Thus, there has been an unfolding panorama of changes in the natural environment as it affects satellite orbit determination.

As the space programs have evolved, numerous decisions were made concerning the design of the satellite and the orbit. A number of these had important effects upon the problem of orbit determination. Some of them are indicated in Figure 4. For example, the original designs called for spherical satellites. Shortly after the space age began, however, long, slender cylindrical satellites were put into orbit. These were tumbling satellites whose orientation was not known. Accordingly, the effective cross-sectional area which they presented to the aerodynamic flow was not known or predictable. This complicated the problem of predicting the satellite's future position. It also rendered more difficult the interpretation of satellite deceleration data in terms of atmospheric density. The Echo satellite had a much lower areal density than any of its predecessors. As a result, even though it was launched into a nearly circular orbit at a height of about 1000 miles, it was significantly affected by atmospheric drag. In addition it was affected in a striking way by

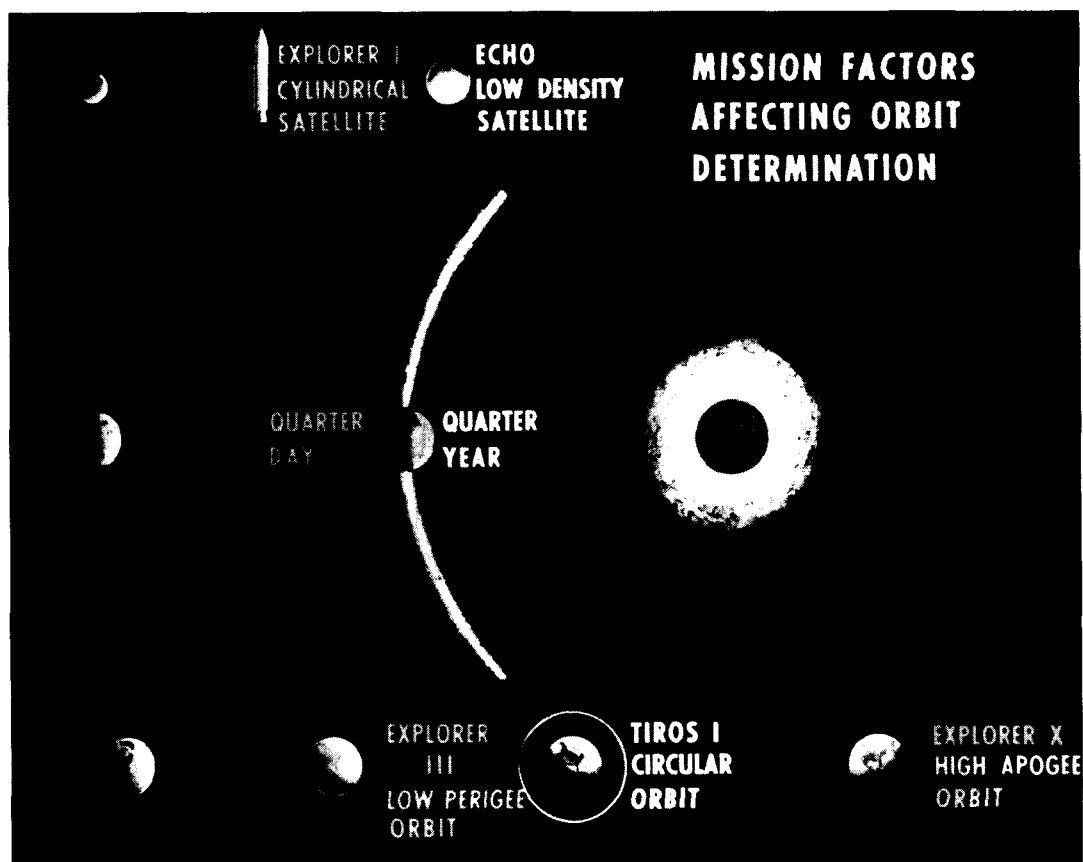


Figure 4—Mission factors affecting orbit determination.

radiation pressure. The radiation pressure effect, barely discernable for ordinary satellites, produced a depression of Echo's perigee of literally several hundreds of miles. The variation of the perigee and apogee heights of Echo due to this effect can be seen in Figure 5.

Many of the orbits themselves varied significantly from the type initially planned for the IGY. For example, the orbit of Explorer III had an unusually low perigee height of about 117 miles. The low perigee of Explorer III resulted in an extremely large drag effect. The period deceleration was of the order of a thousand times greater than that which was encountered in the case of other IGY satellites. The first Tiros satellite, designed to obtain new information about cloud cover and weather, brought new problems from the orbit determination standpoint. The orbit of this satellite was very nearly circular. This meant that certain orbit determination theories, which had been developed originally for the elliptic satellite orbit case, had to be modified appropriately to handle this type of orbit.

Following the initial scientific satellite studies, interest grew in the regions well beyond the immediate neighborhood of the earth. Accordingly, high apogee satellite orbits were planned. Explorer VI, for example, was launched into an orbit having an apogee height of about 24,000 miles. This was followed by Explorer X, whose apogee was at 180,000 miles from the earth, or three-quarters of the way to the moon's orbit. In this satellite, the experimental equipment operated until the satellite reached the neighborhood of apogee on the first orbit, or for a period of approximately two and a half days. Explorer XII was launched into an orbit having an apogee of about 48,000 miles. Its experimental equipment operated for over three months. The orbit of each of these satellites was significantly affected by the lunar and solar perturbations. Accordingly, it was necessary to include the effects of these gravitational forces in determining the orbits of these new high-apogee satellites.

The original IGY plans called for a satellite with an active life of about a fortnight. As the useful lifetimes of satellites increased, and the periods over which their orbits were determined increased correspondingly, it became customary to determine orbits on a weekly basis. This meant that differential corrections were performed each week, using the data covering a one week interval. The orbit determination theories were adequate to handle this type of orbital time span. These separate arcs were discontinuous. The minimum distances between them in the regions where they overlapped were of the same order as the uncertainties in position in the individual arcs, however. Thus, a piece-wise continuous representation of the orbit could be obtained by means of a sequence of week long orbital arcs. For certain purposes, however, the precision obtained in this way was not sufficient. For example, in carrying out certain types of magnetic experiments in satellites it was found to be desirable to obtain a truly continuous representation of

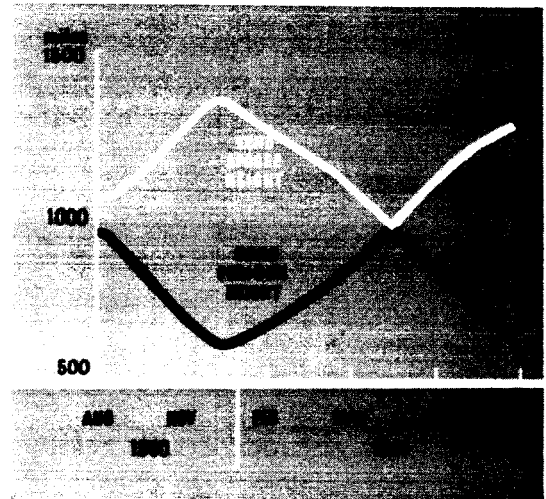


Figure 5—Radiation pressure effect on ECHO orbit determined.

the orbital path over the entire active lifetime of the satellite. This was done in the case of the satellite Vanguard III, for example. In this case the orbital arc considered had a duration of approximately a quarter of a year. New theoretical methods had to be developed to handle this type of case. Some indication of the severity of the problems associated with orbital arcs of this length can be had by recalling that classical astronomy deals with observations of the planets extending back about two centuries. A close earth satellite completes as many orbits in a fortnight as a terrestrial planet does in two centuries. On this basis a quarter of a year for the close earth satellite corresponds to more than a millenium for a terrestrial planet. The difficulties were further increased due to the fact that atmospheric drag force is an important one for artificial satellites, whereas forces of this type are not nearly so significant for the planets and their natural satellites. These new, long orbital arcs required new theoretical developments.

At the other end of the scale, considerable interest has been focused on orbits lasting only a quarter of a day, such as those encountered in Project Mercury, for example. The original Project Mercury mission involved the completion of three orbits of the earth by the astronaut. Some of the theoretical problems associated with long arcs were not present in this case. On the other hand, however, a number of additional problems had to be considered, in view of the fact that the astronaut's safety was at stake. Special measures had to be taken to insure that adequate information concerning the orbit and the trajectory was available at all times from the moment the launching rocket left the pad until the capsule returned to the surface of the earth in the recovery area. In order to insure that adequate supplies of tracking data were available to meet this objective, extensive use has been made of radar tracking systems. A network was established consisting of some ten radars which were capable of tracking the Mercury capsule during its first orbit. The first of these radars tracked the Atlas rocket as it left the launching pad and traversed the satellite launching trajectory. As the satellite capsule was projected into orbit, it could actually be tracked by two such radars. These radars were capable of obtaining ten complete sets of measures of range, azimuth and elevation each second. These radar measures were of high precision. Thus, within less than a minute after the Mercury satellite entered its orbit, sufficient information was available to determine whether, in fact, the satellite actually was in orbit and, further, whether the orbit was satisfactory from the standpoint of the mission objectives. This information had to be available on this time scale in order to permit the making of the vital decision as to whether the mission should be continued, or whether the manned capsule should be brought safely back to earth in an emergency recovery area in the Atlantic Ocean. Once in orbit the Mercury satellite was tracked again and again by means of precision radars, each of which was capable of obtaining, by itself, enough information to permit the determination of an accurate orbit. In all, ten such radars tracked the Mercury capsule during its first orbit, as is indicated in Figure 6. The handling of this tracking information and the making of the critical recommendations concerning go and no-go decisions on a real time basis requires the development of extensive computer program systems. These must be capable of rapid, precise orbit determination. This includes the assessment of the accuracy of the incoming tracking data, the determination of the orbital elements, the prediction of the satellite's position in order to permit acquisition by radars, and the making of critical recommendations involving the go, no-go decisions, the time at which the retro rockets should be fired to cause re-entry into the desired recovery region, etc. These

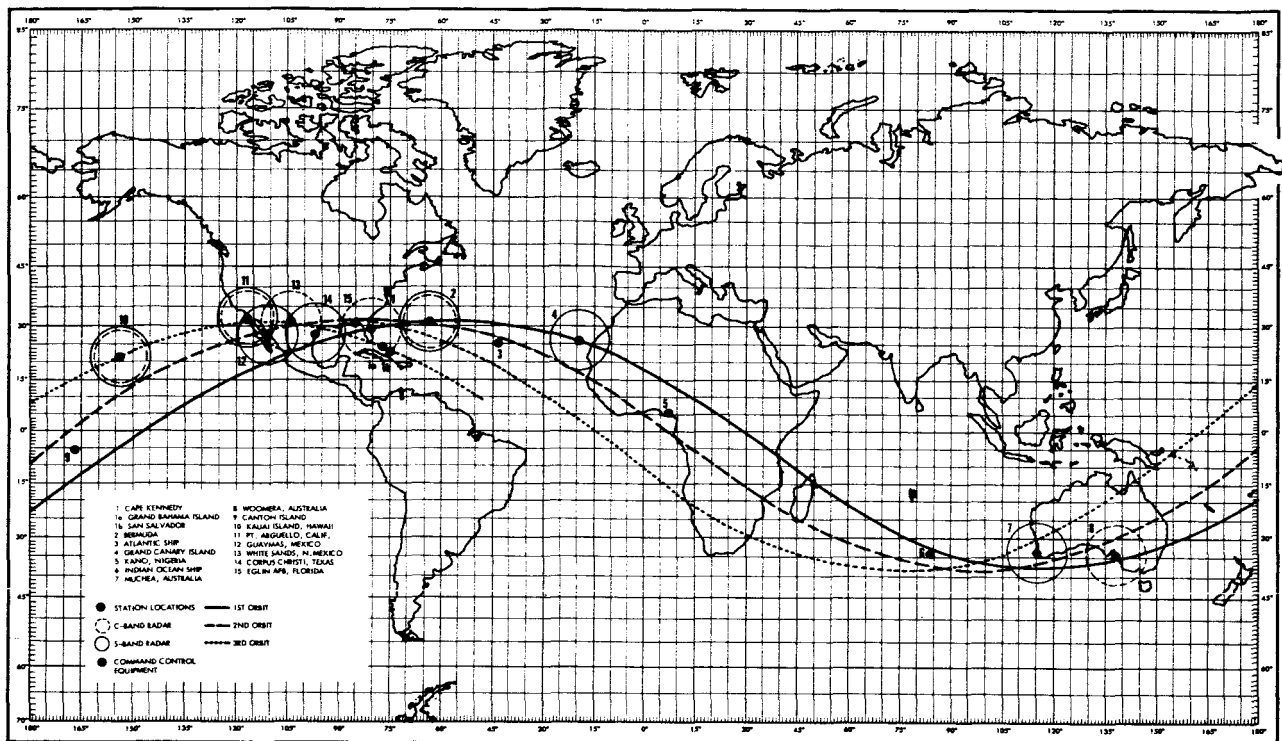


Figure 6—Project Mercury.

programs must all be properly connected in the master, overall Mercury programming system so that they will operate automatically in the real-time situation.

This type of orbit determination operation is in marked contrast to the type conducted for scientific satellites. Scientific satellites are normally tracked by means of the Minitrack system, which operates using the principles of radio interferometry. As a scientific satellite passes over a Minitrack station, measures are obtained giving the direction from the Minitrack station to the satellite. No range information is obtained during such a tracking pass. The Minitrack system utilizes fixed-beam antenna installations. These have several advantages. They do not require special acquisition aids, as do radars of the Mercury type. They will track automatically whenever the satellites enters the antenna beam. The Minitrack system can operate using an extremely light transmitter in the satellite itself. Scientific satellites are generally much smaller than their manned counterparts. The original IGY satellites, for example, weighed about 20 pounds, whereas the Mercury capsules weigh more than a ton, or over a hundred times the weight of the original scientific satellites. In fact, in the Mercury Capsule, the weight of the tracking equipment alone is greater than the entire weight of America's first scientific satellite. Even though the weights involved are different by approximately two orders of magnitude, the Minitrack system for tracking scientific satellites is capable of great precision. In general, however, weight is traded for the rate at which tracking information becomes available to the central computing system. Ordinarily, a scientific satellite is observed roughly once per orbit. Hence, it is necessary to observe the satellite over a time interval corresponding to three or four orbits before reasonably good orbital



information can be obtained. This is in marked contrast to the Mercury tracking system which is capable of providing accurate orbital information on the basis of a single radar pass. These two types of orbit determination operations require quite different approaches from the analytical, mathematical, and computing standpoints.

Still a different array of techniques is used to track high-apogee satellites. Satellite orbits extending out to distances of ten earth radii or so have dynamical properties which are significantly different from those of close-earth satellite orbits. For example, once a satellite is more than a few earth radii from the earth, its rate of angular motion is slow. Accordingly, angular tracking systems are less appropriate here than they are for close-earth satellites. The range and the range rate of a high-apogee satellite are, however, changing appreciably over most of the orbit. Accordingly, it is desirable to measure these quantities. It is for this reason that doppler and ranging systems have been developed for tracking spacecraft travelling out toward the moon and deep into space. The Goddard orbit determination system has utilized data of this type in determining orbits of satellites and spacecraft. A new range and range rate tracking system is presently being developed by Goddard especially for use in tracking high-apogee scientific and applications satellites in the forthcoming OGO and Syncom programs.

The present Goddard orbit determination system now in operation includes the basic elements of the type described earlier. In each of the principal areas, however, there has been a significant increase in the complexity of problems to be considered and a corresponding increase in the complexity of the theories and the computer programs developed for orbit determination. This is indicated in Figure 7. For example, the original system was designed to use Minitrack data primarily. At the present time the system is capable of using many different types of data including not only Minitrack data but also the Mercury radar tracking data referred to earlier, Baker-Nunn precision optical tracking data, data from a number of doppler, range and range rate systems, and data from the Deep Space Information Facility developed by the Jet Propulsion Laboratory of the California Institute of Technology.

In the theoretical area, the original modification of Hansen's theory developed to apply to the close-earth satellite case is now but one of a number of different theoretical methods available for use in the Goddard general orbit determination system. Some of these are indicated in Figure 8. A numerical integration method is also available. Some refinements have been added to this method in order to permit control of the buildup of round-off error. A new method developed by Prof. Brouwer of Yale University is also in extensive use. The method of variation of parameters is also used to handle special problems arising in connection with high-apogee satellites, satellites perturbed significantly by radiation pressure, etc.

The environment now is clearly recognized to be far more complex than had been suspected at the time the space age began. The Goddard orbit determination system now takes into account the many new types of perturbations referred to above. Modifications have also been incorporated to deal with the many types of satellites, the different types of orbits and the various orbit intervals discussed above.

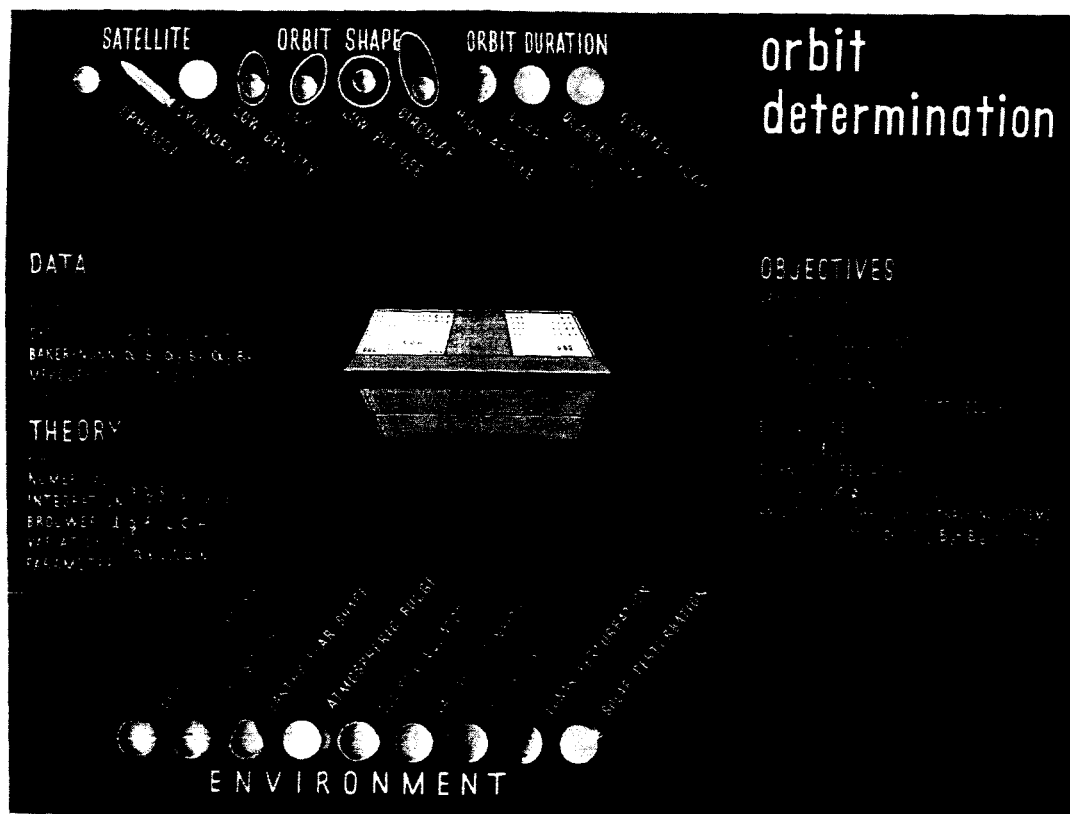


Figure 7—Orbit determination.

Orbit determination objectives have become more numerous in response to the increasing needs of the space programs. The problem of determining initial orbits has grown more complex as the launching trajectories and the orbits themselves have become more varied and complex. Antenna pointing predictions must now be furnished for a wide variety of antenna systems including not only those which are based on direction cosines, azimuth and elevation, and right ascension and declination, but also those which are based on hour angle and declination, prime vertical angle and meridian axis angle, and meridian angle and prime vertical axis angle. Some of these antenna systems are indicated in Figure 9.

It is also necessary to furnish predictions with a much greater degree of precision than was necessary heretofore. For example, in the case of Project Echo, the satellite was tracked by means of large 84-foot dishes operating at frequencies in the neighborhood of 2300 megacycles. The beam width of such

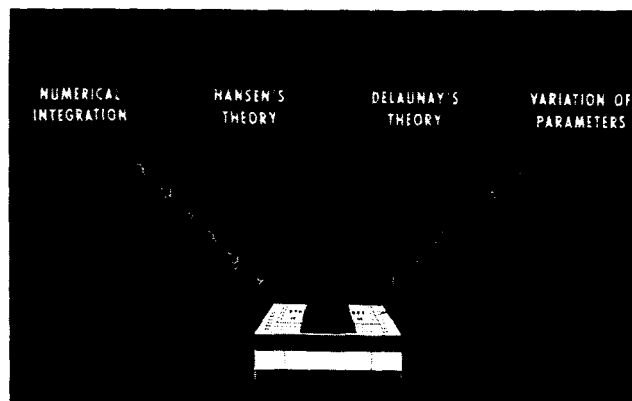


Figure 8—Orbit determination theories.

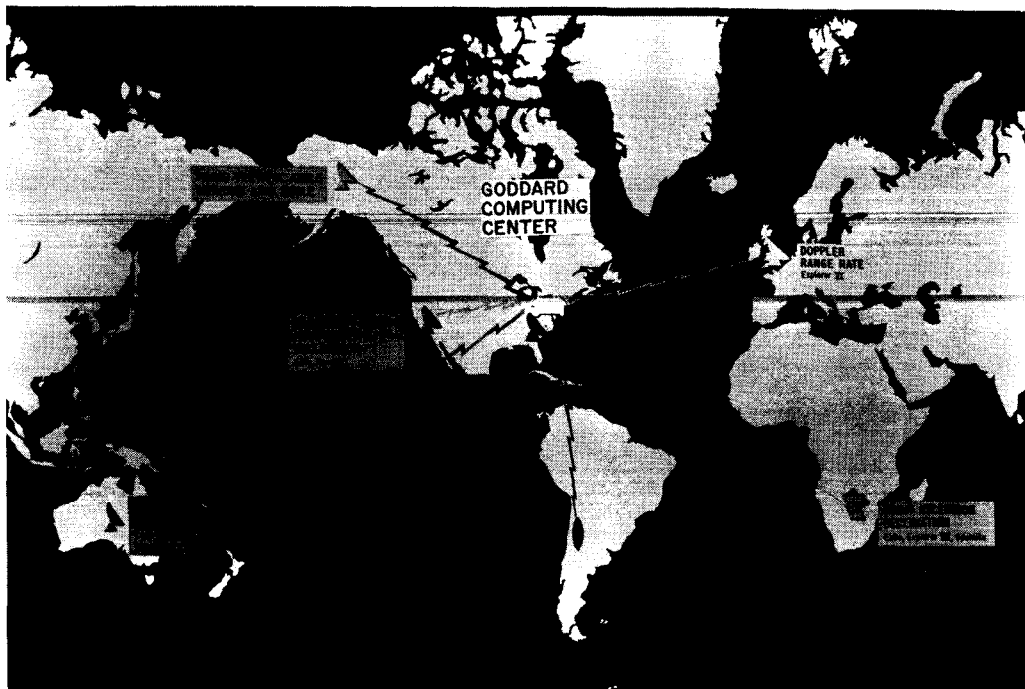


Figure 9—Various types of tracking data.

an antenna system is extremely narrow. The problem was further compounded by the fact that one of the operations involved the simultaneous pointing of two such dishes at the satellite. This mode of operation further decreased the effective beam width of the combined two-antenna system. The beam width of this combined system was approximately 0.15 degrees. Accordingly, it was necessary to furnish pointing predictions with this accuracy. The antenna systems used in Project Echo were not all capable of automatically tracking the satellite. It was necessary actually to point them at the satellite by means of drive tapes generated by the Goddard orbit determination system. These drive tapes were used to point the large antenna systems at the satellite from opposite coasts of the United States. This method was used to transmit messages from President Eisenhower and Senator, now Vice-President, Lyndon B. Johnson across the continent via the Echo satellite. This is indicated in Figure 10. Because of the large and unpredictable effect of atmospheric drag upon the orbit and position of Echo, it was decided to redetermine the orbit each day on the basis of the latest tracking information during critical operations. It was found that it was possible, using this approach, to predict the position of the Echo satellite with an accuracy of 0.15 degrees for periods up to a day in advance. On the same basis it was found possible to predict the position of more dense satellites such as Tiros with an accuracy of 0.1 degrees. Greater accuracy was possible in the case of these latter satellites due to the fact they are perturbed to a much smaller degree by the variable and unpredictable atmospheric drag forces. Since Echo was easily visible to the naked eye, great interest was shown in observing this satellite by people all over the world. It thus was necessary to predict the times at which Echo would be visible from cities around the world. Some of the cities for which Echo predictions have been made are shown in Figure 11.

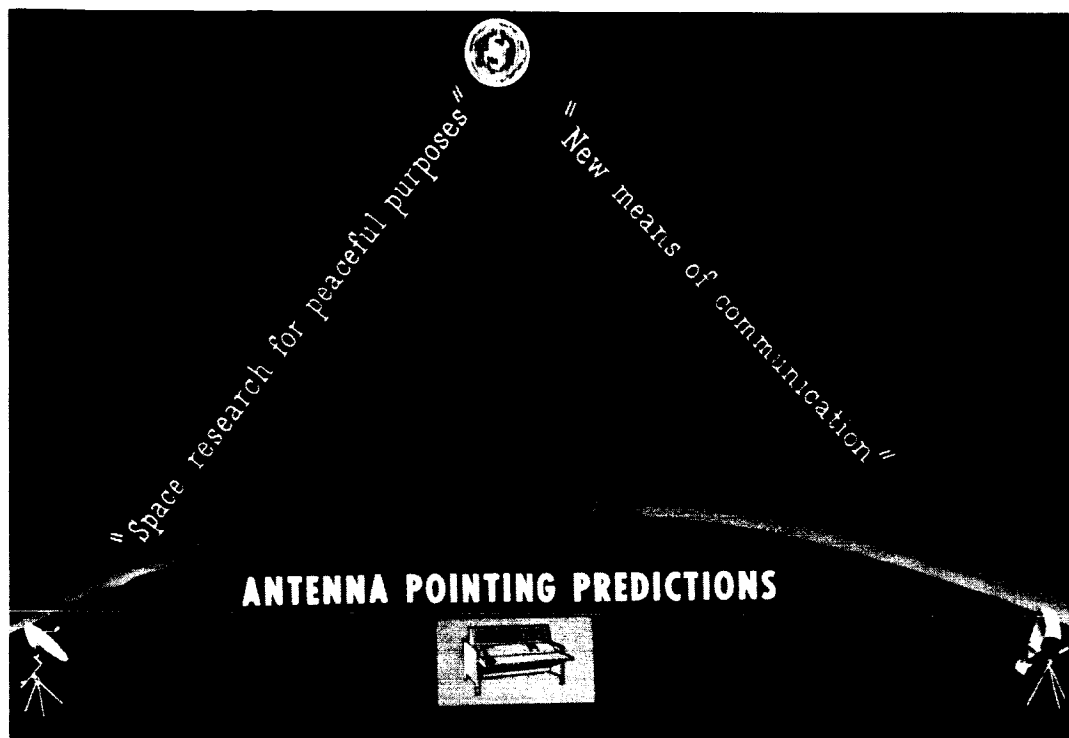


Figure 10—Antenna pointing predictions.



Figure 11—Cities for which Echo predictions were computed.

Many of you may have seen and actually used such predictions which frequently appear in our local newspapers. Predictions of this type are furnished continuously for literally hundreds of cities around the world by the Goddard Computing Center.

Many scientific experimenters are now interested not only in the positions of their satellites in geographic space but also in the magnetic parameters of the satellite environment. In such cases it is customary to give the Greenwich time, the satellite position in terms of geographic latitude, longitude, and height, as well as the magnitude of the magnetic field vector and three components of this vector. In addition, certain other geomagnetic quantities of interest to particular experimenters are determined and furnished along with the other ephemeris information. For example, the coordinates of the satellite in B-L space are provided for experimenters studying the radiation belts, both natural and artificial.

The attitudes of satellites such as Tiros are determined on the basis of data obtained from special horizon sensing equipment mounted in the satellite. Detailed information concerning the attitude history of such satellites is also furnished in addition to the basic ephemeris information for use by certain experimenters.

Scientific research results of a number of different types have been obtained from detailed and definitive analyses of orbital tracking data. The contributions of Vanguard I in this connection, indicated in Figure 12, are especially notable, since this satellite contained no experimental apparatus, per se. All of the Vanguard I research results have been achieved through orbit analyses. New measures of the earth's oblateness were obtained using the Vanguard I orbital data. The earth was discovered to be pear shaped on the basis of Vanguard I orbital information. The atmospheric bulge and the correlation of atmospheric density with solar activity were discovered by means of Vanguard I tracking data. Solar radiation pressure effects were first calculated in a definitive way for the Vanguard I satellite. A new measure of the scale of the solar system itself was obtained through analysis of the doppler observations obtained from Pioneer V. This spacecraft was launched into an orbit around the sun which carried it toward the orbit of Venus as is indicated in Figure 13. It was tracked to the unprecedented distance of more than 20,000,000 miles from the earth. Careful analysis of the doppler tracking data from this satellite yielded a new measure for the value of the astronomical unit, which corresponds to the mean distance of the earth from the sun.

The accuracy with which orbits are determined has steadily increased through the years. At the present time the uncertainty in a satellite's position as determined on the basis of Minitrack observations is of the order of 100 seconds of arc. The moon is approximately 1800 seconds of arc in diameter. Accordingly, the uncertainty of a satellite position in terms of directional or angular information corresponds to about 1/18th of the diameter of the moon. The corresponding uncertainty in satellite position for satellites at a mean distance of the order of a thousand kilometers is of the order of half a kilometer. This is comparable to the accuracies with which the positions of sounding rockets were known during the period before the IGY. The present precision of orbital information represents an increase of approximately an order of magnitude in accuracy over that which was achieved in the early days of the IGY. Analyses and inter-comparisons of different types of tracking

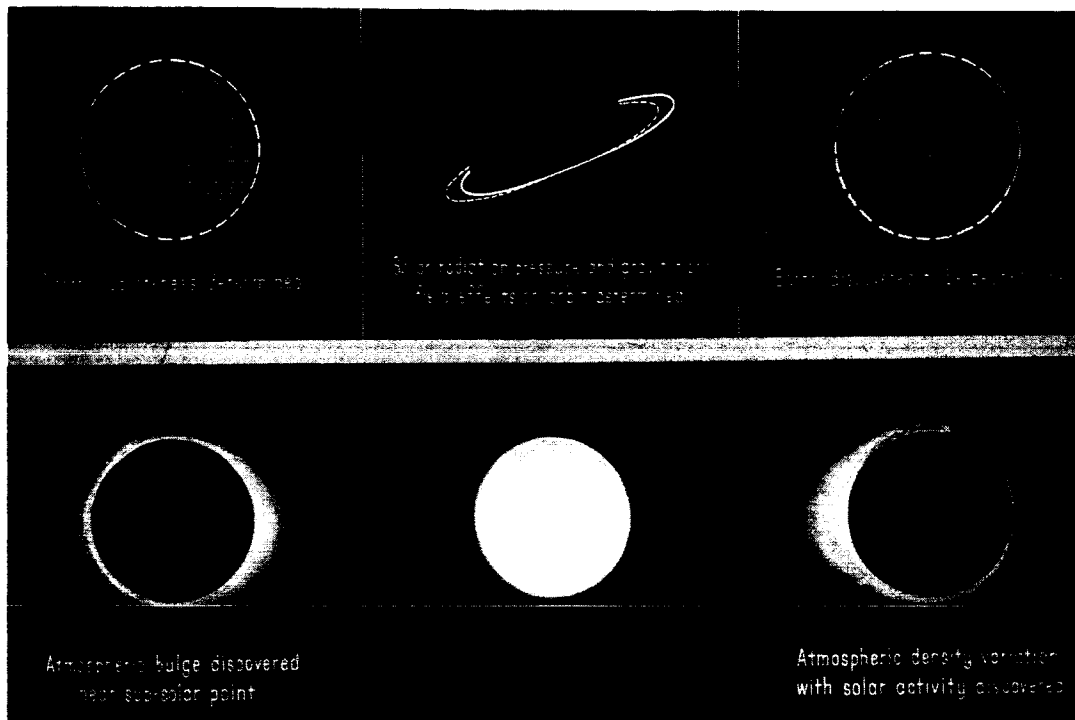


Figure 12—Vanguard I scientific research results from orbital tracking data.

systems have been conducted as new systems came into operation. It has been found, for example, that the Minitrack and Baker-Nunn tracking systems give consistent results.

A number of new, interesting orbit determination problems have arisen in connection with some of the newer NASA programs. In the future, it is planned not only to determine the orbits of satellites and spacecraft but also actually to control and to change them. Project Gemini planners, for example, contemplate the bringing together of two satellites in rendezvous operations. In general the orbits of the two satellites will differ initially. It will be necessary to determine the orbit of each of these satellites precisely. It will then be necessary to determine what changes should be made to the orbits of one or the other or both of the satellites in order to bring the two satellites together. Once the two satellites have been brought reasonably close to

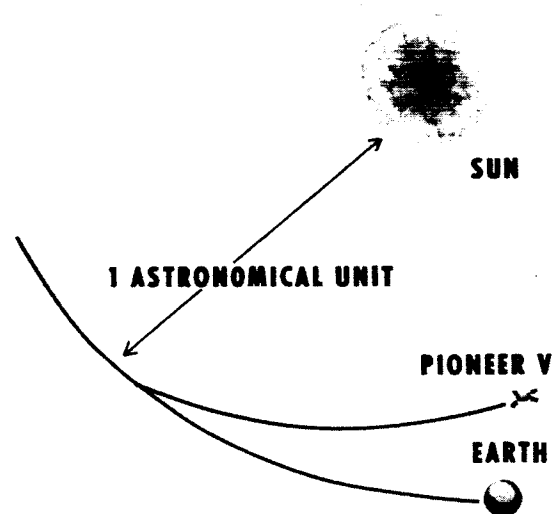


Figure 13—Scientific research results from orbital tracking data—size of solar system determined.

each other, the astronauts will take over and guide the satellites during the terminal phases of the rendezvous.

A number of new theoretical problems arise in connection with orbit determination systems now under development. The unsolved problems of principal interest currently are associated with the perturbative forces indicated in Figures 2 and 14. General perturbation theories have been developed which represent the zonal gravitational harmonics. It is desirable to extend these theoretical developments to treat the other types of perturbations indicated in Figure 14. There is also interest in further development of special perturbation orbit determination systems as they apply to lunar orbits and trajectories. These problems being of a theoretical nature, lend themselves especially well to cooperative efforts involving both NASA centers and universities. NASA is already working closely with university groups in tackling them. NASA and Goddard are looking forward to continued and extended cooperative efforts involving Goddard and educational institutions.

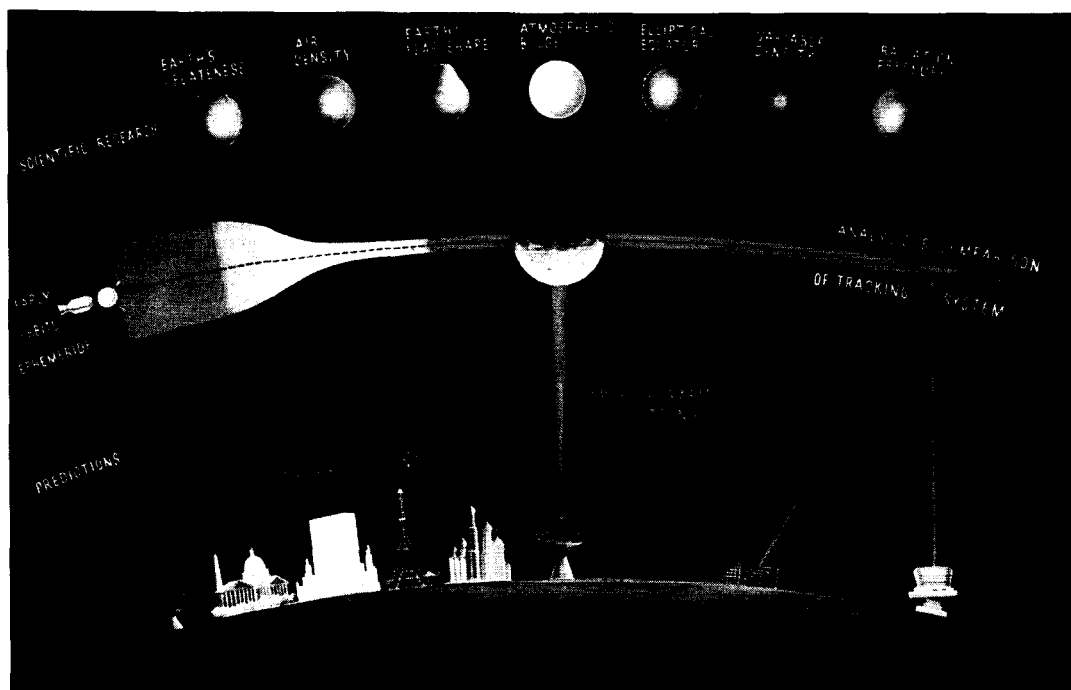


Figure 14—Orbit determination.